

INSTRUMENTATION ENGINEERING

CONTROL SYSTEMS AND PROCESS CONTROL



Comprehensive Theory
with Solved Examples and Practice Questions





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Control Systems and Process Control

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Transfer Function

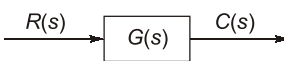
2.1 TRANSFER FUNCTION AND IMPULSE RESPONSE FUNCTION

In control theory, transfer functions are commonly used to characterise the input-output relationships of components or systems that can be described by linear, time-invariant differential equations.

Transfer Function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Transfer Function of Open Loop System :



$$G(s) = \frac{C(s)}{R(s)}$$

Transfer Function of Closed Loop System :

Transfer function of closed loop system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

$R(s)$ = Reference input

$C(s)$ = Controlled output

$E(s)$ = Actuating error signal

$G(s)$ = Forward path transfer function

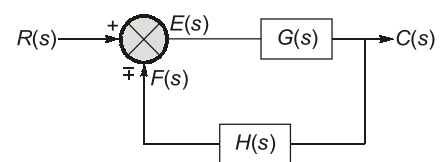
$H(s)$ = Feedback path transfer function

$$C(s) = G(s)E(s)$$

$$= G(s)[R(s) \pm C(s)H(s)] = G(s)[R(s) \pm G(s)C(s)H(s)]$$

$$C(s) \pm G(s)H(s)C(s) = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$



Shortcut Method

1. To find close loop transfer function from open loop transfer function

$$\text{If} \quad \text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}$$

$$\text{Then,} \quad \text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} + \text{Numerator}}$$

2. To find open loop transfer function from close loop transfer function

$$\text{If} \quad \text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}$$

$$\text{Then,} \quad \text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} - \text{Numerator}}$$

Linear Systems

A system is called linear if the principle of superposition and principle of homogeneity apply. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by transferring one input at a time and adding the results. It is the principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered as linear.

Linear Time-Invariant Systems and Linear-Time Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations i.e. constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are function of time are called linear time varying systems. An example of a time-varying control system is a space craft control system (the mass of a space craft changes due to fuel consumption).

The definition of transfer function is easily extended to a system with multiple inputs and outputs (i.e. a multivariable system). In a multivariable system, a linear differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect (on any output) due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone.

EXAMPLE : 2.1

When deriving the transfer function of a linear element

- (a) both initial conditions and loading are taken into account.
- (b) initial conditions are taken into account but the element is assumed to be not loaded.
- (c) initial conditions are assumed to be zero but loading is taken into account.
- (d) initial conditions are assumed to be zero and the element is assumed to be not loaded.

Solution : (c)

While deriving the transfer function of a linear element only initial conditions are assumed to be zero, loading (or input) can't assume to be zero.

EXAMPLE : 2.2

If the initial conditions for a system are inherently zero, what does it physically mean?

- (a) The system is at rest but stores energy
- (b) The system is working but does not store energy
- (c) The system is at rest or no energy is stored in any of its part
- (d) The system is working with zero reference input

Solution : (c)

A system with zero initial conditions is said to be at rest since there is no stored energy.

2.2 STANDARD TEST SIGNALS

1. Step Signal

$$r(t) = A u(t)$$

where, unit step signal $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Laplace transform, $R(s) = A/s$

2. Ramp Signal

$$r(t) = \begin{cases} A t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^2$

3. Parabolic Signal

$$r(t) = \begin{cases} A t^2 / 2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^3$

4. Impulse Signal

$$r(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} ; \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

Laplace transform, $R(s) = 1$

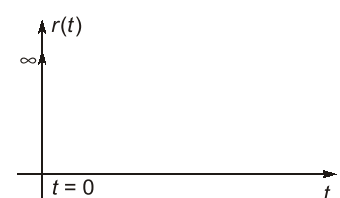
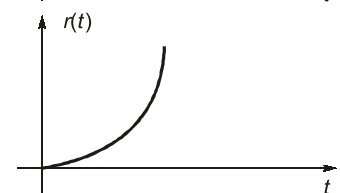
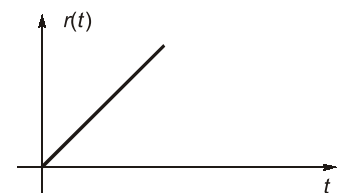
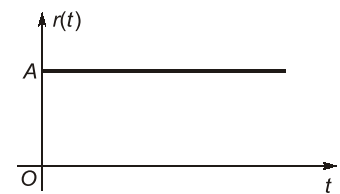
Transfer function, $G(s) = \frac{C(s)}{R(s)}$

$$C(s) = F(s) R(s)$$

Let, $R(s) = \text{Impulse signal} = 1$

$$C(s) = \text{Impulse response} = G(s) \times 1 = \text{Transfer Function}$$

$$\mathcal{L}\{\text{Impulse Response}\} = \text{Transfer function} = \left[\frac{C(s)}{R(s)} \right]$$





- Td/dt (Parabolic Response) = Ramp Response
- d/dt (Ramp Response) = Step Response
- d/dt (Step Response) = Impulse Response

Consider, a linear time-invariant system has the input $u(t)$ and output $y(t)$. The system can be characterized by its impulse response $g(t)$, which is defined as the output when the input is a unit-impulse function $\delta(t)$. Once the impulse response of a linear system is known, the output of the system $y(t)$, with any input $u(t)$, can be found by using the transfer function.

Let $G(s)$ denotes the transfer function of a system with input $u(t)$, output $y(t)$, and impulse response $g(t)$. The transfer function $G(s)$ is defined as

$$G(s) = \mathcal{L}[g(t)] = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} \Big|_{\text{initial conditions} \rightarrow 0} = \frac{Y(s)}{U(s)}$$



REMEMBER

Sometimes, students do a common mistake, they first find $y(t)/u(t)$ and then take its Laplace transform to determine the transfer function which is absolutely wrong. Because,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} \neq \mathcal{L}\left[\frac{y(t)}{u(t)}\right]$$

2.3 POLES AND ZEROS OF A TRANSFER FUNCTION

The transfer function of a linear control system can be expressed as

$$G(s) = \frac{A(s)}{B(s)} = \frac{K(s - s_1)(s - s_2) \dots (s - s_n)}{(s - s_a)(s - s_b) \dots (s - s_m)}$$

where K is known as gain factor of the transfer function $G(s)$.

In the transfer function expression, if s is put equal to $s_a, s_b \dots s_m$ then it is noted that the value of the transfer function is infinite. These $s_a, s_b, \dots s_m$ are called the poles of the transfer function.

In the transfer function expression, if s is put equal to $s_1, s_2 \dots s_n$ then it is noted that the value of the transfer function is zero. These $s_1, s_2 \dots s_n$ are called the zeros of the transfer function.

2.3.1 Multiple Poles and Multiple Zeros

The poles $s_a, s_b \dots s_m$ or the zeros $s_1, s_2 \dots s_n$ are either real or complex and the complex poles or zeros always appear in conjugate pairs.

It is possible that either poles or zeros may coincide; such poles or zeros are called multiple poles or multiple zeros.

2.3.2 Simple Poles and Simple Zeros

Non-coinciding poles or zeros are called simple poles or simple zeros. From the transfer function expression, it is observed that

- If $n > m$, then the value of transfer function is found to be infinity for $s = \infty$. Hence, it is concluded that there exists a pole of the transfer function at infinity (∞) and the multiplicity (order) of such a pole being $(n - m)$.

- If $n < m$, then the value of transfer function is found to be zero for $s = \infty$. Hence, it is concluded that there exists a zero of the transfer function at infinity (∞) and the multiplicity (order) of such a zero being $(m - n)$.

Therefore, for a rational transfer function the total number of zeros is equal to the total number of poles.

The transfer function of a system is completely specified in terms of its poles, zeros and the gain factor.

Consider the following transfer function:

$$G(s) = \frac{s + 3}{(s + 2)(s + 1 + 3j)(s + 1 - 3j)}$$

For the above transfer function, the poles are at :

(a) $s_a = -2$; (b) $s_b = -1 - 3j$; and (c) $s_c = -1 + 3j$

The zeros are at $s_1 = -3$.

As the number of zeros should be equal to number of poles, the remaining two zeros are located at $s = \infty$.

The pole-zero plot is plotted as shown:

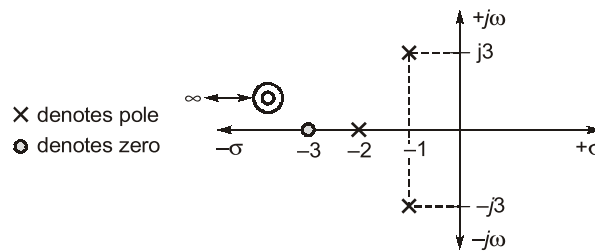


Fig. : Pole-zero plot

Note : Poles and zero are those complex/critical frequencies which make the transfer function infinity or zero.

2.3.3 Proper Transfer Functions

The transfer functions are said to be strictly proper if the order of the denominator polynomial is greater than that of the numerator polynomial (i.e. $m > n$). If $m = n$, the transfer function is called proper. The transfer function is improper if $n > m$.

In the transfer function expression of a control system, the highest power of s in the numerator is generally either equal to or less than that of the denominator.

EXAMPLE : 2.3

A transfer function has two zeros at infinity. Then the relation between the numerator degree (N) and the denominator degree (M) of the transfer function is

(a) $N = M + 2$

(b) $N = M - 2$

(c) $N = M + 1$

(d) $N = M - 1$

Solution : (b)

For a rational transfer function, the total number of zeros are equal to total number of poles.

Therefore, Number of poles = M ; Number of zeros = $N + 2$

For a rational transfer function : $M = N + 2$ or $N = M - 2$

Using equation (i) when input is $u(t)$, output is

$$\frac{H(s+c)}{s(s+a)(s+b)} = \frac{K_1}{s} + \frac{D}{s+a} + \frac{E}{s+b}$$

Taking inverse Laplace transform,

$$\text{Output} = 2 + De^{-t} + Ee^{-3t}$$

So, $a = 1$ and $b = 3$

Using final value theorem

$$\Rightarrow \lim_{s \rightarrow 0} \frac{s \cdot H(s+c)}{s(s+a)(s+b)} = \lim_{t \rightarrow \infty} 2 + De^{-t} + Ee^{-3t}$$

$$\frac{Hc}{ab} = 2 \quad \text{and} \quad Hc = 6$$

Using equation (i) when input is $e^{-2t}u(t)$, output is $\frac{H(s+c)}{(s+2)(s+a)(s+b)}$

Only two terms are present in the response.

Hence $s+c = s+2$

$\Rightarrow c = 2$

$H = 3$

($\because HC = 6$)



OBJECTIVE BRAIN TEASERS

Q1 A control system with certain excitation is governed by the following mathematical equation

$\frac{d^2x}{dt^2} + \frac{1}{2} \frac{dx}{dt} + \frac{1}{18}x = 10 + 15e^{-4t} + 2e^{-5t}$. The natural time constants of the response of the system are

- (a) 2s and 5s (b) 3s and 6s
(c) 4s and 5s (d) $\frac{1}{3}s$ and $\frac{1}{6}s$

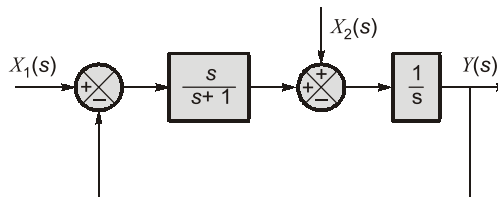
Q2 The response $g(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $g(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

- (a) $(1 + e^{-t} + e^{-2t})u(t)$
(b) $(e^{-t} + e^{-2t})u(t)$
(c) $(1.5 - e^{-t} - 0.5e^{-2t})u(t)$
(d) $e^{-t}\delta(t) + e^{-2t}u(t)$

Q3 The frequency response of a linear time-invariant system is given by $H(f) = \frac{5}{1 + j10\pi f}$. The step response of the system is

- (a) $5(1 - e^{-5t})u(t)$ (b) $5(1 - e^{-t/5})u(t)$
(c) $\frac{1}{5}(1 - e^{-5t})u(t)$ (d) $\frac{1}{(s+5)(s+1)}$

Q4 For the following system :



when $X_1(s) = 0$, the transfer function $\frac{Y(s)}{X_2(s)}$ is

- (a) $\frac{s+1}{s^2}$ (b) $\frac{1}{s+1}$
(c) $\frac{s+2}{s(s+1)}$ (d) $\frac{s+1}{s(s+2)}$

Q5 Ramp response of the transfer function

$$F(s) = \frac{s+1}{s+2} \text{ is}$$

- (a) $\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}t$ (b) $\frac{1}{4}e^{-2t} + \frac{1}{4} + \frac{1}{2}t$
(c) $\frac{1}{2} - \frac{1}{2}e^{-2t} + t$ (d) $\frac{1}{2}e^{-2t} + \frac{1}{2} - t$

Q6 Which of the following statements are correct?

1. Transfer function can be obtained from the signal flow graph of the system.
2. Transfer function typically characterizes to linear time invariant systems.
3. Transfer function gives the ratio of output to input in frequency domain of the system.

- (a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q7 Which of the following is not a desirable feature of a modern control system?

- (a) Quick response
(b) Accuracy
(c) Correct power level
(d) Oscillations

Q8 In regenerating feedback, the transfer function is given by

- (a) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$
(b) $\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1-G(s)H(s)}$
(c) $\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$
(d) $\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$

Q9 The principle of homogeneity and superposition are applied to

- (a) linear time variant systems
(b) non-linear time variant systems
(c) linear time invariant systems
(d) non-linear time invariant systems

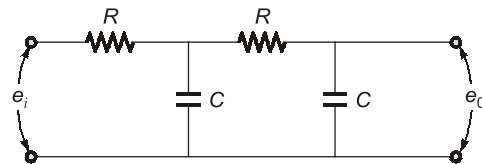
Q10 If a system is represented by the differential equation, is of the form $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = r(t)$

- (a) $k_1e^{-t} + k_2e^{-9t}$ (b) $(k_1 + k_2)e^{-3t}$
(c) $ke^{-3t}\sin(t + \phi)$ (d) $te^{-3t}u(t)$

Q11 A linear system initially at rest, is subject to an input signal $r(t) = 1 - e^{-t}$ ($t \geq 0$). The response of the system for $t > 0$ is given by $c(t) = 1 - e^{-2t}$. The transfer function of the system is

- (a) $\frac{(s+2)}{(s+1)}$ (b) $\frac{(s+1)}{(s+2)}$
(c) $\frac{2(s+1)}{(s+2)}$ (d) $\frac{(s+1)}{2(s+2)}$

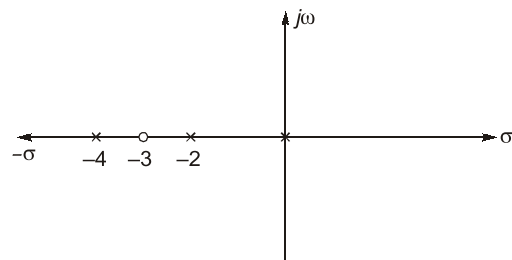
Q12 Consider the RC circuit shown in figure below:



The transfer function $\frac{E_o(s)}{E_i(s)}$ will be ($\tau = RC$)

- (a) $\frac{1}{\tau^2 s^2 + 2\tau s + 1}$ (b) $\frac{1}{\tau^2 s^2 + 3\tau s + 1}$
(c) $\frac{\tau_s}{\tau^2 s^2 + 2\tau s + 1}$ (d) $\frac{\tau_s}{\tau^2 s^2 + 3\tau s + 1}$

Q13 The pole-zero configuration of a transfer function is shown in figure. The value of transfer function at $s = 1$ is found to be 4. Then the transfer function of system is



- (a) $\frac{12(s+3)}{s(s+2)(s+4)}$ (b) $\frac{15(s+3)}{s(s+2)(s+4)}$
(c) $\frac{4(s+3)}{s(s+2)(s+4)}$ (d) $\frac{10(s+2)}{s(s+3)(s+4)}$

Q.14 The transfer function of a system is given by

$$\frac{C(s)}{R(s)} = \frac{100}{(s+10)(s^2+2s+1)}, \text{ using the concept}$$

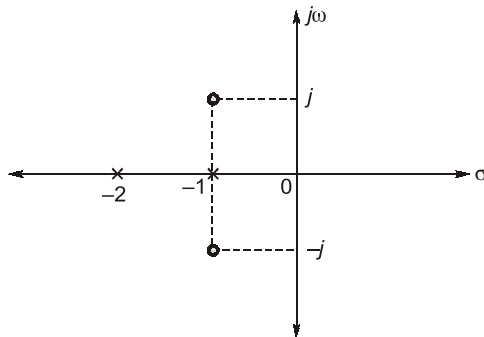
of dominant pole, the 2nd order approximation of above transfer function is

- (a) $\frac{100}{s^2+2s+1}$ (b) $\frac{10}{s^2+2s+1}$
(c) $\frac{10}{s+10}$ (d) $\frac{100}{(s+10)}$

Q.15 A differentiator has a transfer function whose

- (a) Magnitude decreases linearly with frequency
(b) Magnitude increases linearly with frequency
(c) Phase increases linearly with frequency
(d) Phase is constant

Q.16 The pole-zero plot of a system is given below. If $G(s) = 15$ for $s = 2$, then the transfer function of the system is



- (a) $\frac{12(s^2+2s+5)}{(s+1)(s+2)}$ (b) $\frac{18(s^2+2s+2)}{(s+1)(s+2)}$
(c) $\frac{12(s^2+2s+3)}{(s+1)(s+2)}$ (d) $\frac{6(s^2+2s+3)}{(s+1)(s+2)}$

Q.17 A linear time invariant system initially at rest, when subjected to unit step input gives a response of $2te^{-5t}$, $t > 0$, the corresponding transfer function is

- (a) $\frac{2}{s(s+5)^2}$ (b) $\frac{2s}{(s+5)}$
(c) $\frac{2s}{(s+5)^2}$ (d) $\frac{2s}{(s-5)^2}$

ANSWER KEY

1. (b) 2. (c) 3. (b) 4. (d) 5. (a)
6. (d) 7. (d) 8. (d) 9. (c) 10. (d)
11. (c) 12. (b) 13. (b) 14. (b) 15. (b,d)
16. (b) 17. (c)

HINTS & EXPLANATIONS

1. (b)

Natural time constants of the response depend only on poles of the system.

$$\begin{aligned} T(s) &= \frac{C(s)}{R(s)} \\ &= \frac{1}{s^2 + s/2 + 1/18} \\ &= \frac{18}{18s^2 + 9s + 1} \\ &= \frac{1}{(6s+1)(3s+1)} \end{aligned}$$

This is in the form $\frac{1}{(1+sT_1)(1+sT_2)}$

$$\therefore T_1, T_2 = 6 \text{ sec}, 3 \text{ sec.}$$

2. (c)

Transfer function of system is impulse response of the system with zero initial conditions.

$$\text{Transfer function} = G(s) = \mathcal{L}(e^{-t} + e^{-2t})$$

$$= \frac{1}{s+1} + \frac{1}{s+2}$$

$$G(s) = \frac{C(s)}{R(s)} = \left(\frac{1}{s+1} + \frac{1}{s+2} \right)$$

$$\text{For step input, } R(s) = \frac{1}{s}$$

$$\begin{aligned} C(s) &= R(s) \cdot G(s) = \frac{1}{s} \left(\frac{1}{s+1} + \frac{1}{s+2} \right) \\ &= \frac{1}{s(s+1)} + \frac{1}{s(s+2)} \end{aligned}$$

$$C(s) = \left(\frac{1}{s} - \frac{1}{s+1} \right) + \frac{1}{2} \left(\frac{1}{s} - \frac{1}{s+2} \right)$$

$$= \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}$$

$$\text{Response} = c(t) = \mathcal{L}^{-1}[C(s)]$$

$$= \mathcal{L}^{-1} \left[\frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2} \right]$$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})u(t)$$

3. (b)

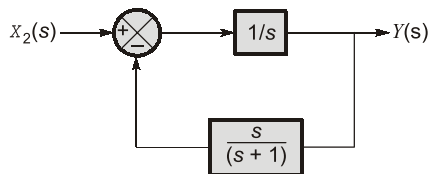
$$H(f) = \frac{5}{1 + j10\pi f}$$

$$H(s) = \frac{5}{1 + 5s} = \frac{5}{5 \left(s + \frac{1}{5} \right)} = \frac{1}{s + \frac{1}{5}}$$

$$\text{Step response} = \frac{1}{s} \cdot \frac{1}{\left(s + \frac{1}{5} \right)}$$

$$Y(s) = \frac{5}{s} - \frac{5}{s + \frac{1}{5}}$$

$$\Rightarrow y(t) = 5[1 - e^{-t/5}] u(t)$$

4. (d)Redrawing the block diagram with $X_1(s) = 0$ 

The transfer function

$$T(s) = \frac{Y(s)}{X_2(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \dots(i)$$

$$\text{Here, } G(s) = \frac{1}{s} \text{ and } H(s) = \frac{s}{s+1}$$

$$\frac{Y(s)}{X_2(s)} = \frac{1/s}{1 + \frac{1}{s} \times \frac{s}{s+1}} = \frac{(s+1)}{s(s+2)}$$

5. (a)

$$\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$$

$$\therefore C(s) = R(s) \cdot \frac{s+1}{s+2}$$

$$= \frac{1}{s^2} \cdot \frac{s+1}{s+2} = \frac{1}{s^2} \left(1 - \frac{1}{s+2} \right)$$

$$= \frac{1}{s^2} - \frac{1}{s^2(s+2)}$$

$$\frac{1}{s^2} \cdot \left(\frac{s+1}{s+2} \right) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$s+1 = As(s+2) + B(s+2) + Cs^2$$

$$= As^2 + 2As + Bs + 2B + Cs^2$$

$$\therefore A + C = 0, 2A + B = 1 \text{ and } 2B = 1$$

$$\therefore A = \frac{1}{2}, B = \frac{1}{4}, C = -\frac{1}{4}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{1}{4s} + \frac{1}{2s^2} + \left(-\frac{1}{4} \right) \frac{1}{s+2}$$

$$= \frac{1}{4}u(t) + \frac{1}{2}tu(t) - \frac{1}{4}e^{-2t}u(t)$$

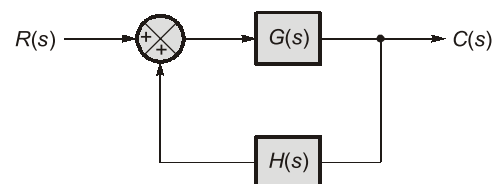
6. (d)

- (i) Transfer function can be obtained from signal flow graph of the system.
- (ii) Transfer function typically characterizes to LTI systems.
- (iii) Transfer function gives the ratio of output to input in s-domain of system.

$$\text{TF} = \frac{L[\text{Output}]}{L[\text{Input}]} \Big|_{\text{Initial conditions} = 0}$$

8. (d)

Block diagram of regenerating feedback system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

10. (d)

Let $R(s)$ is the input Laplace transform of given differential equation is

$$s^2 Y(s) + 6sY(s) + 9Y(s) = R(s)$$

$$(s^2 + 6s + 9) Y(s) = R(s)$$

$$TF = \frac{Y(s)}{R(s)} = \frac{1}{(s+3)^2}$$

$$IR = L^{-1} \left[\frac{1}{(s+3)^2} \right] = te^{-3t} u(t)$$

11. (c)

Given that :

$$r(t) = 1 - e^{-t},$$

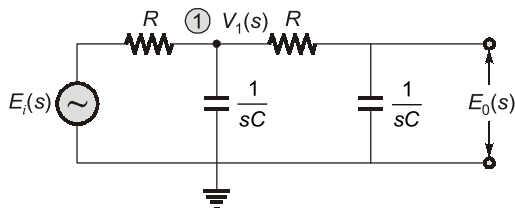
$$c(t) = 1 - e^{-2t}$$

$$R(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$$

$$C(s) = \frac{1}{s} - \frac{1}{(s+2)} = \frac{2}{s(s+2)}$$

$$TF = \frac{C(s)}{R(s)} = \frac{2s(s+1)}{s(s+2)} = \frac{2(s+1)}{s+2}$$

12. (b)



KCL at node-1,

$$\frac{V_1(s)}{1/sC} + \frac{V_1(s)}{R+1/sC} + \frac{V_1(s) - E_i(s)}{R} = 0$$

$$V_1(s) = \frac{E_i(s)/R}{sC + \frac{sC}{1+SCR} + \frac{1}{R}}$$

$$V_1(s) = \frac{E_i(s)(1+SCR)}{sC(1+SCR)R + SCR + 1 + SCR}$$

$$V_1(s) = \frac{E_i(s)(1+SCR)}{S^2 R^2 C^2 + 3SCR + 1}$$

$$E_o(s) = \frac{1/sC}{R + \frac{1}{sC}} \cdot V_1(s)$$

$$E_o(s) = \frac{1}{1+SCR} \times \frac{E_i(s)(1+SCR)}{S^2 R^2 C^2 + 3SCR + 1}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{S^2 R^2 C^2 + 3SCR + 1}$$

$$= \frac{1}{(s^2 \tau^2 + 3\tau s + 1)}$$

13. (b)

Poles location, $s = 0, s = -2, s = -4$

Zero's location, $s = -3$

$$G(s) = \frac{k(s+3)}{s(s+2)(s+4)}$$

$$G(s)|_{s=1} = \frac{k(1+3)}{1 \times (1+2) \times (1+4)} = 4$$

$$\Rightarrow k = \frac{4 \times 5 \times 3}{4} = 15$$

$$\text{Hence, } G(s) = \frac{15(s+3)}{s(s+2)(s+4)}$$

14. (b)

Poles location,

$$s_1 = -10 = P_1$$

$$s_{2,3} = (s^2 + 2s + 1) = (s+1)^2$$

$$= -1, -1 = P_2, P_3$$

$$\text{Since, } \frac{P_1}{P_2} > 5, \frac{P_1}{P_3} > 5$$

$$\therefore \frac{C(s)}{R(s)} = \frac{10}{\left(\frac{s}{10} + 1\right)(s^2 + 2s + 1)}$$

Pole $\left(\frac{s}{10} + 1\right)$ can be neglected, hence 2nd order approximation,

$$\frac{C(s)}{R(s)} = \frac{10}{(s^2 + 2s + 1)}$$

15. (b, d)

The relationship between the input and output of differentiator is given as

$$y(t) = \frac{dx}{dt}(t)$$

\uparrow L.T.

$$Y(s) = sX(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = s$$

Put $s = j\omega$,

$$H(j\omega) = j\omega$$

$$|H(j\omega)| = \omega$$

$$\angle H(j\omega) = 90^\circ \text{ (always)}$$

So, it is constant with w.r.t. frequency.

$$G(s) = 15 = \frac{K(2^2 + 2(2) + 2)}{(2+1)(2+2)}$$

$$\therefore K = \frac{15 \times 3 \times 4}{10} = 18$$

Therefore, transfer function of the system is given as,

$$G(s) = \frac{18(s^2 + 2s + 2)}{(s+1)(s+2)}$$

16. (b)

Poles : $-1, -2$

Zeros : $-1 + j, -1 - j$

\therefore Transfer function is given by,

$$G(s) = \frac{K((s+1)^2 + 1)}{(s+1)(s+2)},$$

where gain is assumed to be K

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s+1)(s+2)}$$

at $s = 2$, $G(s) = 15$

17. (c)

Given : $y(t) = 2te^{-5t}$

$x(t) = u(t)$

Taking Laplace transform, we get,

$$Y(s) = \frac{2}{(s+5)^2} \quad \text{and} \quad X(s) = \frac{1}{s}$$

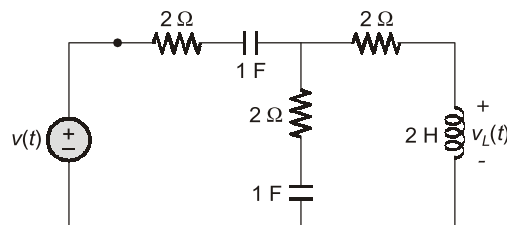
\therefore Overall transfer function,

$$\frac{Y(s)}{X(s)} = \frac{2s}{(s+5)^2}$$



CONVENTIONAL BRAIN TEASERS

Q1 Determine the transfer function, $G(s) = \frac{V_L(s)}{V(s)}$ for the network shown below using mesh analysis.



1. (Sol.)

For the given network assuming loop current as shown in figure.

Writing mesh equation using KVL,

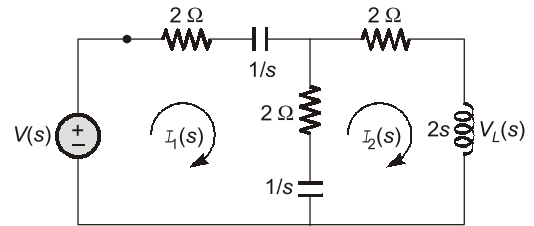
$$-V(s) + 2I_1(s) + \frac{I_1(s)}{s} + (I_1(s) - I_2(s)) \left(2 + \frac{1}{s} \right) = 0$$

$$V(s) = \left(4 + \frac{2}{s} \right) I_1(s) - \left(2 + \frac{1}{s} \right) I_2(s) \quad \dots(1)$$

Similarly, $(2 + 2s)I_2(s) + \left(2 + \frac{1}{s}\right)(I_2(s) - I_1(s)) = 0$

$$\left(4 + 2s + \frac{1}{s}\right)I_2(s) - \left(2 + \frac{1}{s}\right)I_1(s) = 0$$

$$-\left(2 + \frac{1}{s}\right)I_1(s) + \left(4 + 2s + \frac{1}{s}\right)I_2(s) = 0 \quad \dots(2)$$



Eliminating $I_1(s)$ using equation (2),

$$I_1(s) = \frac{\left(4 + 2s + \frac{1}{s}\right)I_2(s)}{\left(2 + \frac{1}{s}\right)} = \left(\frac{4s + 2s^2 + 1}{2s + 1}\right) \cdot I_2(s)$$

Substituting equation (1),

$$V(s) = \left(4 + \frac{2}{s}\right)\left(\frac{4s + 2s^2 + 1}{2s + 1}\right)I_2(s) - \left(2 + \frac{1}{s}\right)I_2(s)$$

$$V(s) = \frac{4s + 2}{s}\left(\frac{4s + 2s^2 + 1}{2s + 1}\right)I_2(s) - \left(\frac{2s + 1}{s}\right)I_2(s)$$

$$V(s) = \frac{2}{s}(4s + 2s^2 + 1)I_2(s) - \frac{(2s + 1)}{s}I_2(s)$$

$$V(s) = \frac{(8s + 4s^2 + 2 - 2s - 1)}{s}I_2(s) = \frac{4s^2 + 6s + 1}{s}I_2(s)$$

$$\frac{I_2(s)}{V(s)} = \frac{s}{4s^2 + 6s + 1}$$

$$2s \frac{I_2(s)}{V(s)} = \frac{2s^2}{4s^2 + 6s + 1} = \frac{V_L(s)}{V(s)}$$

■■■■